AP calculus BC   Summer 2023 Review

Hello and welcome to AP calc BC 2023-2024!
Below you will find a set of questions covering most of the topics you learned in AP calculus AB.

Your task is to answer the following questions (all your work should be on separate paper)

1736, 1739, 1740, 1741, 1742, ,1748, 1750-1759, 1762, 1768-1770, 1773-1780, 1785, 1790
1800-1815, 1834-1842, 1883, 1884, 1886, 1888, 1892, 1895
1901 (not part D), 1911, 1917, 1924, 1925, 1927, 1935

On the second class (in September) you will take a quiz made exclusively with questions from this list.

You may want to wait until mid-summer!
You want these skills to be relatively fresh in your mind in the fall. Also, don't fake your way through these problems; instead, visit Khanacademy (they have a well defined AP calculus AB course page) or any other online resource.
The whole packet, when done the right way, will take about two weeks (~10 hrs) to complete.
I see this practice as an opportunity for you to review what you learned last year (look things up, watch video lessons...). In addition, throughout the year we will continue to review AB content any time we need to.

Have a great summer!

Mr. Malossini

Important!!
Most of these questions do not require a calculator! The few questions that require a calculator are labeled "(calculator)".

9.1 Algebra

For use anytime after Section 1.15

Solve each of the following for y.

1736. $x^2 + y^2 = 9$

1737. $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$

1738. $\dfrac{(x-1)^2}{8} + \dfrac{(y+1)^2}{4} = 1$

1739. $\ln y = x^2 + 5$

1740. $e^y = x + 7$

1741. $xy - y + x - 1 = 1$

Find all real zeros of the following functions.

1742. $y = 2x(x-1)^2$

1743. $y = x^3 + 5x^2 + 4x + 20$

1744. $y = x^3(x+2)$

1745. $y = \ln(x-5)\sqrt{x^3 - 8}$

For each pair of functions, find $f(g(x))$ and its domain, then find $g(f(x))$ and its domain.

1746. $f(x) = |x|$; $g(x) = x^2 - 1$

1747. $f(x) = 3x - 4$; $g(x) = \dfrac{x+4}{3}$

1748. $f(x) = \dfrac{1}{x^2} + 1$; $g(x) = \dfrac{1}{x-1}$

1749. $f(x) = x^2 + 1$; $g(x) = \sqrt{x}$

The table below gives values of two functions f and g. Using the table, determine the values of the following compositions.

x	-4	-3	-2	-1	0	1	2
$f(x)$	$-\frac{1}{2}$	$-\frac{2}{3}$	-1	-2	undefined	2	1
$g(x)$	0	-2	0	2	2	0	-2

1750. $f(g(-1))$

1751. $g(f(2))$

1752. $f(g(0))$

1753. $g(f(-1))$

1754. $f(g(-3))$

1755. $f(f(1))$

1756. $g(g(-4))$

1757. $g(g(0))$

9.2 Derivative Skills

For use anytime after Section 2.15

Find the derivative of each function in simplest factored form.

1758. $T(x) = e^x(2x^2+3)$

1759. $h(x) = (5^x)(2^{x-1})$

1760. $o(x) = \ln\left[\dfrac{(x+1)^{16}(2x^2+x)^8}{\sqrt{x^2+4}}\right]$

1761. $m(x) = \ln\left[\dfrac{(e^{2x}+6)^7\sqrt{x+4}}{(e^{-x}+e^x)^5}\right]$

1762. $a(x) = \ln\sqrt{\dfrac{2-3x}{x+4}}$

1763. $s(x) = \dfrac{\log x}{3x}$

1764. $P(x) = (4x-1)\sec 3x$

1765. $o(x) = \dfrac{\tan 3x}{\tan 2x}$

1766. $l(x) = \dfrac{\cos 2x}{\log x}$

1767. $s(x) = x(1+\cos^2 x)$

1768. $t(x) = 3^{x^2}$

1769. $r(x) = \log\left(\dfrac{x+1}{x^2+1}\right)$

1770. $a(x) = 2xe^{-x}$

Find the derivative implicitly.

1771. $2x\ln y - 3y\ln x = 6xy$

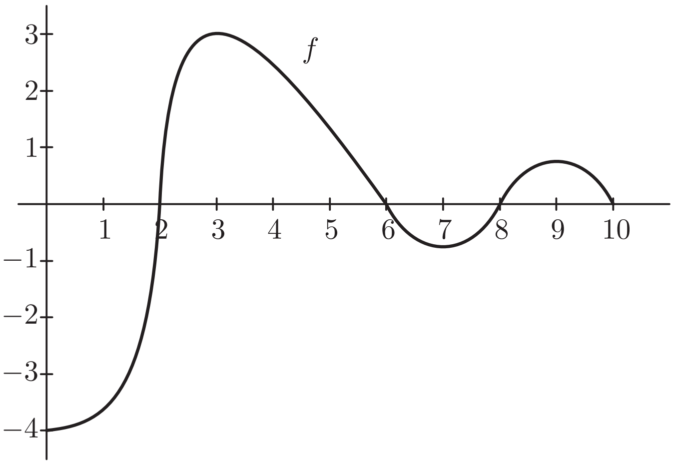
1772. $e^{3y} - 2 = \ln(x^2-4y)$

1773. $xe^{5y} - yx^2 = \ln x$

All economical and practical wisdom is an extension or variation of the following arithmetical formula: $2+2=4$. Every philosophical proposition has the more general character of the expression $a+b=c$. We are mere operatives, empirics, and egotists, until we learn to think in letters instead of figures. —*Oliver Wendell Holmes*

9.3 Can You Stand All These Exciting Derivatives?

For use anytime after Section 3.8



1774. The graph above is the graph of the derivative of a function f. Use the graph to answer the following questions about f on the interval $(0, 10)$. Justify your answers.

a) On what subinterval(s) is f increasing?

b) On what subinterval(s) is f decreasing?

c) Find the x-coordinates of all relative minima of f.

d) Find the x-coordinates of all relative maxima of f.

e) On what subinterval(s) is f concave up?

f) On what subinterval(s) is f concave down?

g) Find the x-coordinates of all points of inflection of f.

1775. F, F , G, and G have values as listed in the table below. Let $P(x) = F(G(x))$, $K(x) = F(x)/G(x)$, $T(x) = F(x)G(x)$, $R(x) = F(x) + G(x)$, and $N(x) = (G(x))^3$. Use the table to evaluate the derivatives.

x	$F(x)$	$F\ (x)$	$G(x)$	$G\ (x)$
1	2	−4	−3	6
−3	1	5	−2	−1

a) $P\ (1)$ b) $K\ (-3)$ c) $T\ (1)$ d) $R\ (-3)$ e) $N\ (-3)$

1776. The line tangent to the graph of a function $g(x)$ at the point $(-5, 4)$ passes through the point $(0, -1)$. Find $g\ (-5)$.

If you must be dull, at least have the good sense to be brief. —*Anonymous*

1777. Let $y = -3x + 2$ be the tangent line to $F(x)$ at $x = 1$. Find $F(1)$ and $F\ (1)$.

1778. The line tangent to the graph of H at $x = 3$ has slope -4 and has an x-intercept at $x = 6$. Find $H(3)$ and $H\ (3)$.

1779 (Calculator). Let $F\ (x) = 3\cos(x) - e^x$.

a) Graph $F\ (x)$ in the window $-5 < x < 2$, $-10 < y < 10$.

b) On what intervals is $F(x)$ increasing on $[-5, 2]$?

c) Where is $F(x)$ concave down on $[-5, 2]$?

d) Where does $F(x)$ have a relative maximum on $[-5, 2]$?

e) Find all inflection points of $F(x)$ on $[-5, 2]$.

f) Sketch a possible graph of $F(x)$ on $[-5, 2]$.

1780. A particle is moving along a line with position function $s(t) = 3 + 4t - 3t^2 - t^3$.

a) Find the velocity function.

b) Find the acceleration function.

c) Describe the motion of the particle for $t \geq 0$.

1781. The positions of two particles on the x-axis are $x_1 = \cos t$ and $x_2 = \cos(t + \frac{\pi}{4})$. What is the farthest apart the particles ever get? When do the particles collide?

1782. The circumference of the equator of a sphere is measured as 10 cm with a possible error of 0.4 cm. This measurement is then used to calculate the radius. The radius is then used to calculate the surface area and volume of the sphere. Estimate the percentage error in the calculated values of a) the radius, b) the surface area, and c) the volume.

1783. Graph the function $a(x) = |x - 1| + |x - 3|$.

a) What is the domain of $a(x)$? Is $a(x)$ even, odd, or neither?

b) Find an expression for $a\ (x)$. *Hint:* $a\ (x)$ is piecewise.

c) Find all relative extrema of $a(x)$.

1784. For groups of 80 or more, a charter bus company determines the rate per person according to the following formula:

$$\text{Rate} = \$8.00 - \$0.05(n - 80)$$

for $n \geq 80$. What number of passengers will give the bus company maximum revenue?

Truth is whatever survives the cleansing fires of skepticism after they have burned away error and superstition.
—*Oliver Wendell Holmes*

9.4 Different Differentiation Problems

For use anytime after Section 3.8

1785. The temperature of an object at time t is given by $T(t) = 0.1(t^4 - 12t^3 + 2000)$ for $0 \le t \le 10$.

a) Find the hottest and coldest temperature during the interval $[0, 10]$.

b) At what time is the rate of change in the temperature a minimum?

1786. Consider the two ellipses given by

$$x^2 + xy + y^2 = 1 \qquad \text{and} \qquad x^2 - xy + y^2 = 1.$$

The first ellipse passes through the points $(1, -1)$ and $(-1, 1)$; the second passes through $(1, 1)$ and $(-1, -1)$. The ellipses intersect in four points: $(0, 1)$, $(1, 0)$, $(0, -1)$, and $(-1, 0)$.

a) Graph the ellipses.

b) Find dy/dx for each ellipse.

c) Find the slope at each intersection point.

d) Find all points on each ellipse where the tangent line is horizontal.

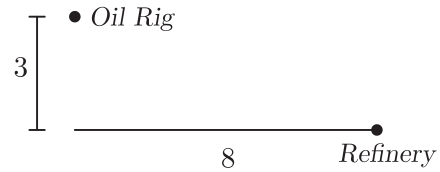
e) Find all points on each ellipse where the tangent line is vertical.

f) Find the two lines that intersect both ellipses at right angles.

1787. Jay is a waiter at a fine-dining restaurant with 100 tables. During his first month he waited on 20 tables every night, and collected an average tip of \$15 from each table. He started to work more tables, and noticed that for every extra table he took on in a night, his average tip would go down 25 cents per table. He figures that he is physically capable of waiting on up to 30 tables in a night. If Jay wants to maximize his tip money, how many more tables should he wait on?

1788. A truck traveling on a flat interstate highway at a constant rate of 50 mph gets 8 miles to the gallon. Fuel costs \$2.30 per gallon. For each mile per hour increase in speed, the truck loses a fifth of a mile per gallon in its mileage. Drivers get \$27.50 per hour in wages, and fixed costs for running the truck amount to \$12.33 per hour. What constant speed should a dispatcher require on a straight run through 260 miles of Kansas interstate to minimize the total cost of operating the truck?

1789. Oil from an offshore rig located 3 miles from the shore is to be pumped to a refinery location on the edge of the shore that is 8 miles east of the rig. The cost of constructing a pipe along the ocean floor from the rig to shore is 1.5 times as expensive as the cost of constructing the pipe on land. How far to the west of the refinery should the pipe come on to shore in order to minimize cost?



1790. Let $f(x) = \arctan x$.

a) Find $f\ (x)$.

b) Evaluate $\lim\limits_{x\to\infty} f(x)$, $\lim\limits_{x\to-\infty} f(x)$, $\lim\limits_{x\to\infty} f\ (x)$, and $\lim\limits_{x\to-\infty} f\ (x)$.

c) Is f even, odd, or neither?

d) Show that f is increasing over all real numbers.

e) Where is f concave up? Concave down? Where are the inflection points, if any exist, of f?

f) How do the graphs of f and f help to confirm your answers?

1791. During Dr. Garner's days as a student last century, he often studied calculus in a dim unheated room with only one candle for light and heat. One particular day in mid-winter, after walking 10 miles (uphill both ways!) through knee-deep snow to attend class, he returned home too tired to study. After lighting the solitary candle on his desk, he walked directly away cursing his woeful situation. The temperture (in degrees Fahrenheit) and illumination (in percentage of candle-power) decreased as his distance (in feet) from his candle increased. In fact, he kept a record of this and in the table below is that information, just in case you may not believe the preceding sad tale!

Distance (feet)	*Temperature* (°F)	*Illumination* (% candle-power)
0	55.0	100
1	54.5	88
2	53.5	77
3	52.0	68
4	50.0	60
5	47.0	56
6	43.5	53

Assume that I get cold when the temperature is below 40°F and it is dark when the illumination is at most 50% of one candle-power.

a) What is the average rate at which the temperature is changing when the illumination drops from 77% to 56%?

b) I can still read my old unlit analog watch when the illumination is 64%. Can I still read my watch when I am 3.5 feet from the candle? Explain.

c) Suppose that at 6 feet the instantaneous rate of change of the temperature is -4.5°F per foot and the instantaneous rate of change of the illumination is -3% candle-power per foot. Estimate the temperature and the illumination at 7 feet.

d) Am I in the dark before I am cold or am I cold before I am in the dark? Explain.

9.5 Integrals... Again!

For use anytime after Section 4.9

Evaluate.

1792. $\displaystyle\int \frac{4y}{3y^2+2}\,dy$

1793. $\displaystyle\int \frac{3z-4z^2+1}{\sqrt{z}}\,dz$

1794. $\displaystyle\int 3^{5y}\,dy$

1795. $\displaystyle\int \sin 7x\,dx$

1796. $\displaystyle\int x\sqrt[4]{3x^2-5}\,dx$

1797. $\displaystyle\int \sin^2 4x\,dx$

1798. $\displaystyle\int e^{\sin 2x}\cos 2x\,dx$

1799. $\displaystyle\int \sin 4x\cos^2 4x\,dx$

1800. $\displaystyle\int (\cos x - 2\sin x)\,dx$

1801. $\displaystyle\int \frac{3x-x^3+1}{x^4}\,dx$

1802. $\displaystyle\int \frac{dx}{x}$

1803. $\displaystyle\int \frac{dx}{2x-3}$

1804. $\displaystyle\int \cos(4x-5)\,dx$

1805. $\displaystyle\int \tfrac{1}{3}x(3x^2-2)^4\,dx$

1806. $\displaystyle\int 2^{3y^2}y\,dy$

1807. $\displaystyle\int \frac{\cos x}{\sin x - 3}\,dx$

1808. $\displaystyle\int \tan 2x\,dx$

1809. $\displaystyle\int \frac{e^{1/x}}{5x^2}\,dx$

1810. $\displaystyle\int \frac{e^{2x}}{e^{2x}-7}\,dx$

1811. $\displaystyle\int \left(\frac{x^2}{3}-\frac{2x^4}{5}-\frac{3}{7}\right)dx$

1812. $\displaystyle\int (e^{2x}+3)^5 e^{2x}\,dx$

1813. $\displaystyle\int \sqrt{x}(x^{1/3}-x^{2/5})\,dx$

1814. $\displaystyle\int e^{\ln x^3}\,dx$

1815. $\displaystyle\int e^{5\sin x}\cos x\,dx$

1816. $\displaystyle\int (3x-2)\sqrt[5]{x}\,dx$

1817. $\displaystyle\int (x^2+3x-2)^3(2x+3)\,dx$

1818. $\displaystyle\int \frac{x^2}{x^3-2}\,dx$

1819. $\displaystyle\int x^3\sqrt{x^4-2}\,dx$

1820. $\displaystyle\int \quad x\sqrt{x}+3\sqrt{x} \quad dx$

1821. $\displaystyle\int \left(\frac{\sqrt[3]{y}}{3}-\frac{3}{\sqrt[3]{y}}\right)dy$

1822. $\displaystyle\int \quad 2x^{1/2}+3x^{-1/2} \quad dx$

1823. $\displaystyle\int 5x\sqrt[3]{(x^2+1)^2}\,dx$

1824. $\displaystyle\int (5x-4)^5 x\,dx$

9.6 Intégrale, Integrale, Integraal, Integral

For use anytime after Section 4.9

Find antiderivatives of the following.

1825. $\int \sec(2u)\tan(2u)\ du$

1826. $\int \cos^2(7u)\ du$

1827. $\int \cot(3x)\ dx$

1828. $\int e^{3x}(e^{3x}-5)^5\ dx$

1829. $\int x\sqrt[3]{3x^2-1}\ dx$

1830. $\int e\ dx$

1831. $\int (3u^2-2u^{-1/3})\ du$

1832. $\int e^{\cos(3x)}\sin(3x)\ dx$

1833. $\int \frac{2}{x+3}\ dx$

1834. $\int 3^{2a}\ da$

1835. $\int 5\cos(5x)\ dx$

1836. $\int \sin(4x)\ dx$

1837. $\int \frac{2x^2+3x-2}{x}\ dx$

1838. $\int \frac{\sec^2 x}{5+\tan x}\ dx$

1839. $\int 3x\sqrt[3]{3x^2-2}\ dx$

1840. $\int (8z+16)^{11}\ dz$

1841. $\int \sqrt{x+2}\ dx$

1842. $\int \sin(6y)\ dy$

1843. $\int \cos(2x)\ dx$

1844. $\int \sec(2x)\ dx$

1845. $\int \cos(4x)\sin^5(4x)\ dx$

1846. $\int \cot a\ da$

1847. $\int 2\cos(2x)\ dx$

1848. $\int 2x\sqrt{x-3}\ dx$

1849. Using what you know about the derivatives of functions like e^x, $\ln x$, and $\sin x$, find functions which satisfy the following equations. For parts (c) and (d), find two different functions.

a) $y \quad - y = 0$ b) $y \quad + y = 0$ c) $y \quad + y = 0$ d) $y \quad - y = 0$

Attaching significance to invariants is an effort to recognize what, because of its form or colour or meaning or otherwise, is important or significant and what is only trivial or ephemeral. A simple instance of failing in this is provided by the poll-man at Cambridge, who learned perfectly how to factorize a^2-b^2 but was floored because the examiner unkindly asked for the factors of p^2-q^2. —*H. W. Turnbull*

9.7 Calculus Is an Integral Part of Your Life

For use anytime after Section 4.9

EVALUATE.

1850. $\displaystyle\int \frac{4y}{(3y^2+2)^5}\,dy$

1851. $\displaystyle\int xe^{5x^2}\,dx$

1852. $\displaystyle\int \cos(5\theta)\,d\theta$

1853. $\displaystyle\int \cos^2(5\theta)\,d\theta$

1854. $\displaystyle\int \cos(5x)\sin^2(5x)\,dx$

1855. $\displaystyle\int \frac{2x}{5x^2-3}\,dx$

1856. $\displaystyle\int \frac{1}{3x}\,dx$

1857. $\displaystyle\int \sin(5\theta-3\pi)\,d\theta$

1858. $\displaystyle\int 4^x\ln 5\,dx$

1859. $\displaystyle\int x\cot(3x^2)\,dx$

1860. $\displaystyle\int \frac{e^{2x}-7}{e^{2x}}\,dx$

1861. $\displaystyle\int 3^x\sin(3^x)\,dx$

1862. $\displaystyle\int (1-\sin^4\theta)\cos\theta\,d\theta$

1863. $\displaystyle\int \frac{1}{x-3}\,dx$

1864. $\displaystyle\int \frac{5}{2-3x}\,dx$

1865. $\displaystyle\int \frac{4y}{2-3y^2}\,dy$

1866. $\displaystyle\int \frac{5z^2}{1+2z^3}\,dz$

1867. $\displaystyle\int \frac{3z-2}{3z^2-4z}\,dx$

1868. $\displaystyle\int \frac{2\sin(3\theta)}{1+\cos(3\theta)}\,d\theta$

1869. $\displaystyle\int \frac{2\cos(4\phi)}{3-\sin(4\phi)}\,d\phi$

1870. $\displaystyle\int \frac{(\ln x)^4}{x}\,dx$

1871. $\displaystyle\int \frac{(\ln x)^3}{x}\,dx$

1872. $\displaystyle\int \sin^6 x\cos x\,dx$

1873. $\displaystyle\int \sin(3x)\cos^2(3x)\,dx$

1874. $\displaystyle\int \sin^2(4x)\,dx$

1875. $\displaystyle\int \sin^3(5x)\,dx$

1876. $\displaystyle\int \sin^3(2x)\cos^3(2x)\,dx$

1877. $\displaystyle\int \tan^2(2x)\,dx$

1878. $\displaystyle\int e^{2x}\tan^2(e^{2x})\,dx$

1879. $\displaystyle\int \frac{1}{2x}\,dx$

1880. $\displaystyle\int_1^{10} \frac{10}{x^2}\,dx$

1881. $\displaystyle\int_1^4 2^x\,dx$

1882. $\displaystyle\int_2^3 \frac{(\sqrt{x}+4)^{-1/2}}{2\sqrt{x}}\,dx$

1883. $\displaystyle\int_4^5 \frac{1}{(y+1)^5}\,dy$

1884. $\displaystyle\int_1^2 10e^x\,dx$

1885. $\displaystyle\int_{5/2}^{13/2} \frac{10}{(2x-4)^{3/2}}\,dx$

1886. $\displaystyle\int_0^1 2e^{5x}\,dx$

1887. $\displaystyle\int_0^5 e^{-0.25x}\,dx$

1888. $\displaystyle\int_1^2 14\sqrt{3x-1}\,dx$

1889. $\displaystyle\int_{-1}^0 2e^{-0.05x}\,dx$

1890. $\displaystyle\int_{-1}^1 \frac{5x}{(15+2x^2)^5}\,dx$

All right, let's do it! —*Grant Wallace*

9.8 Particles

For use anytime after Section 4.14

In the following problems, $s(t)$ is position, $v(t)$ is velocity, and $a(t)$ is acceleration. Find both the net distance and the total distance traveled by a particle with the given position, velocity, or acceleration function.

1891. $v(t) = e^{3t}$, where $0 \leq t \leq 2$

1892. $s(t) = e^{-t+t^2}$, where $0 \leq t \leq 4$

1893. $v(t) = \cos 2t$, where $0 \leq t \leq \pi$

1894. $a(t) = 4t - 6$, where $0 \leq t \leq 2$ and $v(0) = 4$

Find the average value of each function over the given interval.

1895. $H(x) = x^2 + x - 2$; $[0, 4]$

1896. $g(x) = 3e^{3x}$; $[\ln 2, \ln 3]$

1897. $R(x) = \sin x$; $[0, 2\pi]$

1898. $T(x) = \tan 2x$; $[0, \frac{\pi}{8}]$

We speak of invention: it would be more correct to speak of discovery. The distinction between these two words is well known: discovery concerns a phenomenon, a law, a being which already existed, but had not been perceived. Columbus discovered America: it existed before him; on the contrary, Franklin invented the lightning rod: before him there had never been any lightning rod.

Such a distinction has proved less evident than it appears at first glance. Torricelli has observed that when one inverts a closed tube on a mercury trough, the mercury ascends to a certain determinate height: this is a discovery; but in doing this, he has invented the barometer; and there are plenty of examples of scientific results which are just as much discoveries as inventions. —*Jacques Hadamard*

9.9 Areas

For use anytime after Section 4.16

In the following eight problems, find the area under the curve on the interval $[a, b]$ by using
a) a right-hand Riemann sum on n equal subintervals;
b) a left-hand Riemann sum on n equal subintervals;
c) 2 trapezoids on equal subintervals;
d) Simpson's rule with 2 parabolas on equal subintervals; and
e) a definite integral.

1899. $y = 8 - 3x;\ \ [0, 2];\ \ n = 2$

1900. $y = x^2;\ \ [0, 2];\ \ n = 5$

1901. $y = 3x^2 + 5;\ \ [1, 4];\ \ n = 3$

1902. $y = 7;\ \ [-2, 6];\ \ n = 4$

Find the exact area of the region bounded by the given curves.

1903. $y = 25 - x^2,\ y = 0$

1904. $y = \sqrt{x-2},\ y = 0,\ x = 3$

1905. $y = \dfrac{2}{x},\ y = 0,\ x = 1,\ x = e$

1906. $y = \cos 2x,\ y = 0,\ x = 0,\ x = \frac{\pi}{4}$

1907. $y = (x-1)(x-2)(x-3),\ y = 0$

1908. $y = x^2;\ y = x + 6$

1909. $y = x^3 + 1;\ y = 9;\ x = 0$

1910. $y = e^{3x};\ y = 8;\ x = \ln 3$

1911. $y = \tan x;\ y = 1;\ x = 0$

1912. $y = 2x + 10;\ y = x^2 + 2;\ x = 0;\ x = 4$

1913. $y = x^3 - 3x - 3;\ y = 5;\ x = -2;\ x = 2$

1914. $y = (x+2)^{3/2};\ y = -x;\ x = 1;\ x = 2$

1915. $y = x;\ y = x^2$

1916. $y = x;\ y = x^3$

1917. $y = x^2;\ y = x^3$

1918. $y = \sin x;\ y = \cos x;\ y = 0$

1919. $y = x^2 - 1;\ y = 1 - x^2$

1920. $y = x^2 - 2x - 3;\ y = x - 4$

1921. $y = x^2 - x - 15;\ y = 10 - x;\ y = 0$

1922. $y = e^x\sqrt{e^x - 1};\ y = 0;\ x = 0;\ x = \ln 9$

1923. $y = \csc x \cot x;\ y = 0;\ x = \frac{\pi}{6};\ x = \frac{\pi}{3}$

9.10 The Deadly Dozen

For use anytime after Section 5.2

1924. Find $\dfrac{d}{dx}\displaystyle\int_{2x}^{-3} \sqrt{5t-3}\, dt$.

1925. If $F(x) = \displaystyle\int_{7}^{x} \dfrac{\ln(3\pi t)}{t}\, dt$, then find $F\ (x)$.

1926. Find the net distance and the total distance traveled by a particle from time $t = 1$ to $t = 3$ if the velocity is given by $v(t) = -3t^2 + 7t - 2$.

1927. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\, dx = ?$

1928. $\displaystyle\int \frac{1}{x \ln x}\, dx = ?$

1929. Let R be the region bounded by the x-axis, the y-axis, the line $x = 4$ and the curve $y = 2e^{3x}$. Find the area of R.

1930. Find the area bounded by the lines $y = x$, $x = e$ and the hyperbola $xy = 7$.

1931. Find the area bounded by the x-axis and the curve $y = 3x^3 - 6x^2 - 3x + 6$.

1932. Find the average value of $G(x) = \dfrac{\ln x}{x}$ over the interval $[e, 2e]$.

1933. Find the area of the region bounded by the curve $y = x^2 - 3x^2 - 4x$ and the line $y = 0$.

1934. Find the average value of the function $g(x) = 3\sqrt{x}$ over the interval $[0, 2]$.

1935. Let R be the region bounded by $g(x) = 2/x$, $x = 1$, $x = 2$, and $y = 0$.

a) Approximate the area of R by using a left-hand Riemann sum with 2 subintervals.

b) Use 2 trapezoids to approximate the area.

c) Find the exact area of R.

d) Find the volume of the solid generated by revolving R about the x-axis.

e) Find the volume of the solid generated by revolving R about the y-axis.

f) Set up an expression involving an integral that represents the perimeter of R.

g) Use your calculator to evaluate the expression in part (f).

Mathematics is not only real, but it is the only reality. That is that entire universe is made of matter, obviously. And matter is made of particles. It's made of electrons and neutrons and protons. So the entire universe is made out of particles. Now what are the particles made out of? They're not made out of anything. The only thing you can say about the reality of an electron is to cite its mathematical properties. So there's a sense in which matter has completely dissolved and what is left is just a mathematical structure. —*Martin Gardner*