Name _____

Students Entering AP Calculus AB/BC, summer 2023 Packet

When you go into AP calculus, we assume you have certain mathematical skills, especially in algebra. If you do not refresh these skills, you will find that you will consistently get problems incorrect, even though you understand the calculus concepts. It is frustrating for students when they are tripped up by the algebra and not the calculus. This summer packet is intended for you to brush up and possibly relearn these topics.

Wait until mid-summer!

You want these skills to be relatively fresh in your mind in the fall. In addition, do not fake your way through these problems, and <u>don't use your graphing calculator</u>!
The whole packet, when done the right way, will take about two weeks (~12 hrs) to complete.
I see this practice as an opportunity for you to review what you learned last year (look things up, watch video lessons...). In addition, throughout the year we will continue to review pre-calculus content any time we need to.
The first quiz of the year will be made exclusively with questions on this packet.
Have a great summer!
Mr. Malossini

This practice is divided in four sections:

1. Simplify expressions
2. Evaluate functions
3. Solve equations
4. Basic functions and operations

Based on my experience, most trouble comes from **trigonometry and logarithms**.

Answer all questions <u>without a calculator</u>!! [After all, more than half of the AP exam is without a calculator]

It is crucial that you <u>do not</u> use a calculator for any of these questions.

1. Simplify each expression

a. $\dfrac{x^3 - 9x}{x^2 - 7x + 12}$

b. $2\ln(x + 3) - \ln(x)$

c. $\dfrac{x^3}{x^{-5}}$

d. $\dfrac{\dfrac{1}{x} - \dfrac{1}{5}}{\dfrac{1}{x^2} - \dfrac{1}{25}} =$

e. $\dfrac{4 - x}{x^2 - 16}$

f. $\dfrac{x}{\sqrt{x}}$

g. $\dfrac{1}{x + h} - \dfrac{1}{x}$

h. $x^{\frac{3}{2}}\left(x + x^{\frac{5}{2}} - x^2\right)$

i. $\dfrac{2}{\sqrt{3}}$

j. $\dfrac{a^{-1}}{a^{-2}\sqrt{a}}$

k. $sin^2x + cos^2x$

l. $1 - sec^2x$

m. $sin^4x - cos^4x$

n. e^{3lnx}

o. $tanx\ cosx$

p. $\dfrac{sin(2x) - sinx}{2cosx - 1}$

q. $2ln\left(\sqrt{1 - x}\right)$

r. $\sqrt{2x - 3} - \dfrac{1}{\sqrt{2x - 3}}$

Important: you must know all the basic trig identities as well as the double angle identities.

2. Evaluate each quantity and write in simplest form

$sin(\pi)$	$cos(\pi)$	$tan(\pi)$
$sin\left(\frac{\pi}{3}\right)$	$cos\left(\frac{\pi}{6}\right)$	$tan\left(-\frac{3\pi}{4}\right)$
$sin\left(\frac{4\pi}{3}\right)$	$cot\left(\frac{\pi}{3}\right)$	$csc\left(\frac{5\pi}{4}\right)$
$cos\left(-\frac{\pi}{3}\right)$	$cos\left(sin^{-1}\left(\frac{1}{2}\right)\right)$	$sin^{-1}\left(sin\,\frac{\pi}{7}\right)$
$sec\left(-\frac{2\pi}{3}\right)$	$cos\left(\frac{7\pi}{4}\right)$	$sin\left(\frac{5\pi}{2}\right)$
$csc\left(\frac{5\pi}{6}\right)$	$cos^{-1}\left(tan\,\frac{\pi}{4}\right)$	$tan\left(\frac{2\pi}{3}\right)$
$e^{ln\,5}$	$ln\,1$	$27^{\frac{2}{3}}$
$ln\,e^{7}$	$\sqrt[3]{8}$	$16^{\frac{3}{4}}$
e^{0}	$ln\left(\sqrt{e}\right)$	$1^{4.2}$

Important: you must be familiar with the unit circle and especially how to use it to find the exact value of trig functions

3. Solve each equation/inequality for *x*. Give exact values only. Solve trig equations on $[0, 2\pi]$

a. $5^{x+1} = 25$

b. $\frac{1}{3} = 3^{2x+2}$

c. $\log_2 x = 3$

d. $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

e. $\frac{2}{x+1} = \frac{x-2}{2}$

f. $x^2 - 6x + 9 = 0$

g. $\frac{1}{x} + x = 4$

h. $\frac{5}{e^x + 1} = 1$

i. $\sqrt{x-1} - \frac{5}{\sqrt{x-1}} = 0$

j. $3cosx - 1 = 2$

k. $2sin(2x) - \sqrt{3} = 0$

l. $tan(x) \cdot (2cos^2 x - 1) = 0$

m. $secx = 2$

n. $ln(x+1) = 2$

o. $ln\left(e^x\right) = 4$

p. $lnx + 2lnx = 0$

q. $ln\,6 + ln(x) - ln\,2 = 3$

r. $4xy - 2y^2 = 2x$

s. $e^{2x-5} = 1$

t. $\frac{x^4 - 1}{x^3} = 0$

u. $(x+3)(x-3) > 0$

v. $|x - 3| \leq 6$

w. $x^3 + 4x^2 - 5x < 0$

x. $\frac{x+2}{x} \geq 0$

y. $|5x - 2| \geq 25$

z. $\sqrt{x+3} = x - 9$

Note on inequalities:

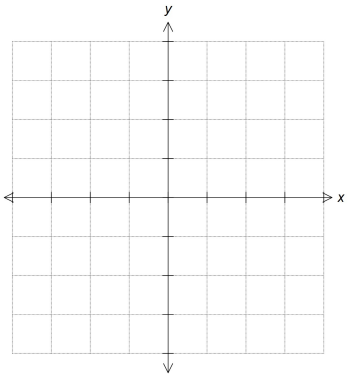
There are a few good videos on Khanacademy. Search for “quadratic inequalities” and “absolute value inequalities”, and then you can adapt the process to similar problems.

4. Basic functions: graphs and operations

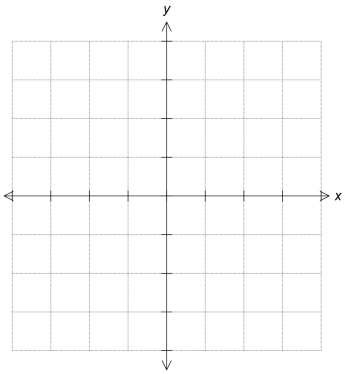
Calculus is a lot easier when you are familiar with several basic functions.
For each function listed below, you should be able to quickly and accurately:

- sketch its graph
- identify domain, range, intercepts, asymptotes, end-behavior …
- perform basic transformations (shifts, stretches, reflections)

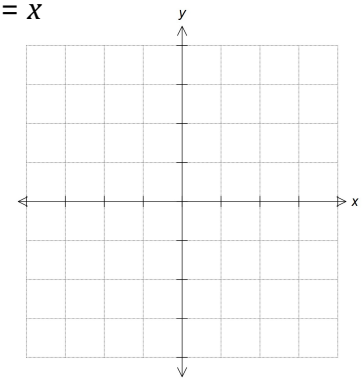
a. $y = \sqrt{x}$



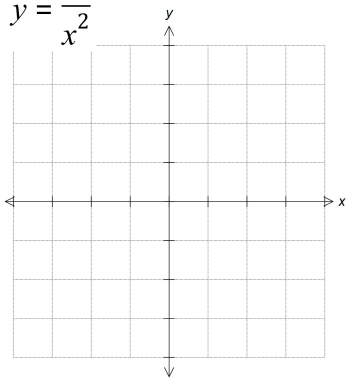
b. $y = \sqrt[3]{x}$



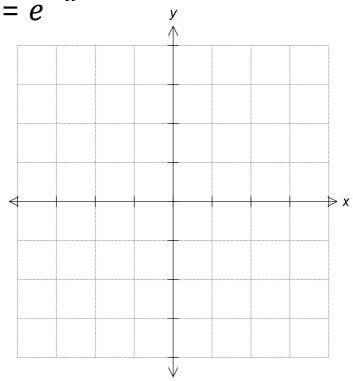
c. $y = x^2$ $\quad y = x^3$



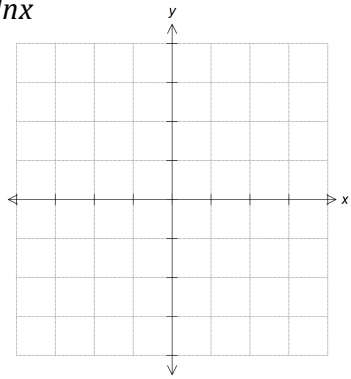
d. $y = \frac{1}{x}$ $\quad y = \frac{1}{x^2}$



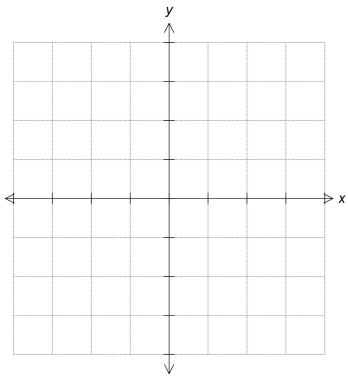
e. $y = e^x$ $\quad y = e^{-x}$



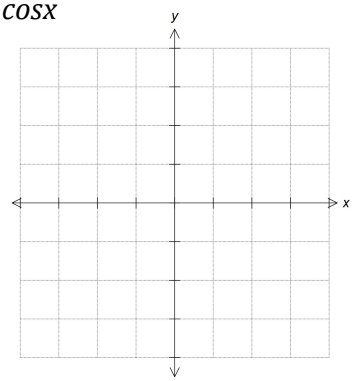
f. $y = \ln x$



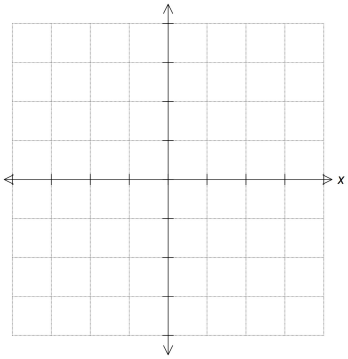
g. $y = \sin x$



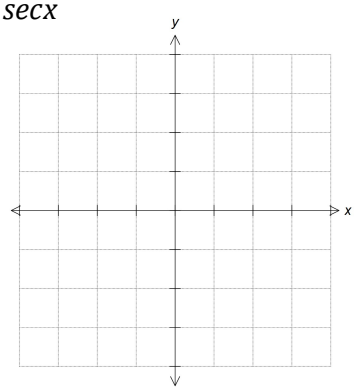
h. $y = \cos x$



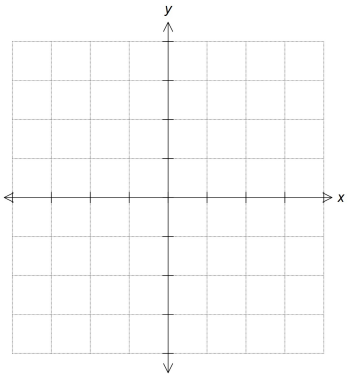
i. $y = \tan x$ and $y = \tan^{-1} x$



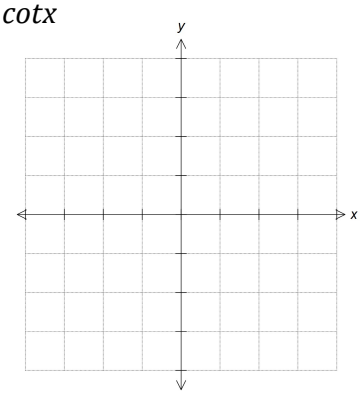
j. $y = \sec x$



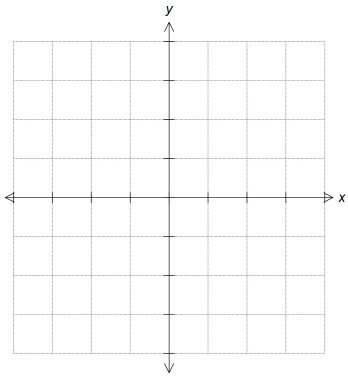
k. $y = \csc x$



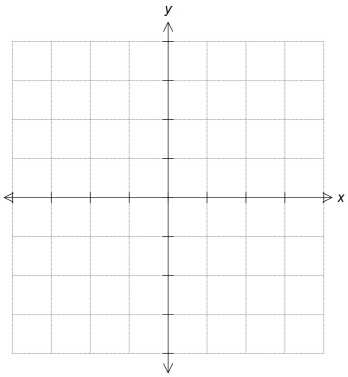
l. $y = \cot x$



m. $y = \begin{cases} \sin x & x < 0 \\ x^2 & x \geq 0 \end{cases}$



n. $y = \begin{cases} x + 1 & x < -1 \\ x^2 - 1 & -1 < x < 1 \\ 2 - x & x \geq 1 \end{cases}$



Operations with functions

o. Find the inverse of each function (is $f^{-1}(x)$ a function? Why? Why not?)

$f(x) = 2\sqrt{3x - 1}$ $\qquad$ $f(x) = \dfrac{3}{x + 2}$ $\qquad$ $f(x) = x^2 + 1$

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p. Given $f(x) = \sqrt{x + 1}$ *and* $g(x) = 3x^2 - 2x$ evaluate and simplify the following:

$g(a + b) =$

$f(x - 3) =$

$f(g(x)) =$

$g(f(x)) =$

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q. Linear functions: find the equation of a line given each condition

- The line with slope 2 passing through (-3,1)
- The line parallel to $3x - y + 1 = 0$ and passing through the origin
- The line that forms a 45° angle with the x-axis and passes through (1,2)
- The line that is perpendicular to $y = \frac{1}{4}x + 1$ and is passing through (3,0)
- The line passing through points (-1,1) and (3,-1)
- The line with slope -2 that forms a triangle of area 12 with the positive x- and y-axis

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r. Find the domain of each function

$y = \sqrt{x + 5}$ $\qquad$ $y = \dfrac{3x}{x - 1}$ $\qquad$ $y = 2^{3 - x}$

$y = ln(x + 2)$ $\qquad$ $y = tan(2x)$ $\qquad$ $y = cos(\pi x)$

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s. Show work to determine if each relation is even, odd, or neither.

$f(x) = 2x^2 - 1$

$f(x) = -x^3 + 2x$

$f(x) = x - \frac{1}{x}$

$f(x) = x^2 + 2x - 4$

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t. Find the equation of each vertical and horizontal asymptote (if any)

$f(x) = \frac{x + 2}{2x - 1}$

$f(x) = \frac{3x}{x^2 + x - 2}$

$f(x) = \frac{x^3 + 5x^2}{x^2 + 9}$

$f(x) = \frac{x + 4}{x^3 - 4x}$

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u. Part 1: Identify the parent graph, then describe and sketch each transformation together with the parent graph.
Part 2: Find the interval/s where (the function) y >0

$y = \sqrt{x - 2} + 3$

$y = sin(2x)$

$y = \left|x^2 - 4\right|$

$y = \frac{1}{|x|}$

$y = (x - 5)^2 - 2$

$y = ln(-x)$

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